**A Level Further Mathematics**

**Transition Activity**

# The questions below are an extension to the A Level Maths transition questions, therefore students taking Double Maths you should complete both the Maths AND Further Maths Transition activities.

# The material in this activity covers the following:

# Topics that students will be familiar with but with questions that are more challenging and aimed at 7-9 grade GCSE students.

# Enrichment topics to extend a student’s overall knowledge and add some interesting puzzles

# An introduction to some of the Further Mathematics topics that a student will be studying in your first year of the Further Mathematics course. Below is the text book which will be required for the course and from which students can follow up with more questions.

# *Pearson Edexcel AS and A Level Further Mathematics*

# *Core Pure Mathematics: Book 1/AS ISBN: 978-1-292-18333-6*

# Additional resources that you may find interesting:

# +plus magazine (<http://plus.maths.org/content/>) for interesting articles on application of mathematics e.g.

# [The maths of infectious diseases](http://plus.maths.org/content/do-you-know-whats-good-you-0#epidemiology):

# Constructing our lives: the mathematics of engineering

# Mathematics and the nature of reality

# Enriching mathematics site (<http://nrich.maths.org/public/>) which has a wide range of puzzles and articles

# There are many interesting popular maths books, here are just a few examples:

# ‘Professor Stewart's Cabinet of Mathematical Curiosities’ by Ian Stewart **ISBN**-10: 1846680646

# ‘Fermat's Last Theorem: The story of a riddle that confounded the world's greatest minds for 358 years’ by Simon Singh **ISBN**-10: 1841157910

# The Penguin Dictionary of Curious and Interesting Numbers (Penguin Press Science) **ISBN**-10: 0140261494

**Section A: Equations and Expressions**

# Simultaneous equations

# 5x + 4y = 23 6x – 7y = 4 Find x and y

# x = 3y + 5 3x – 4y = 26 Find x and y

# x2 + y2 = 29 y – x = 3 Find x and y

# x + 2y – z = 9 2x + 6y – 3z = 23 Find x and express z in terms of y

# x + y + z = 10 2x + y – z = 2 3x + z = 11 Find x, y and z

# sin(x + y) = 1 cos(x – y) = 0.5 Find x and y if 0 ≤ (x + y), (x – y) ≤ 90

# If 5x - y = 18 and 5y - x = 12, What is the value of x - y?

# Anna is twice as old as her brother Carl. Their brother Bob is 3 years older than Carl. 6 years ago Anna’s age was the product of her brothers’ ages. How old are the siblings now if they all have different ages?

#

**Equations**

# Solve the following equations by factorising

# $ x² + 22 x + 72 = 0$

# $x² - 10x - 119 = 0$

# $2x² - 3x - 5 = 0$

# $x³ + 10x² + 21x = 0 $

# $$5. \frac{7}{x+1}+ \frac{6}{x-1}=6$$

# Expressions

# Simplify the following expression by expanding the brackets and collecting like terms

# $$\left(x+y\right)\left(x-y\right)$$

# Simplify the following expression by expanding the brackets and collecting like terms

# $$\left(2x + 3y\right)^{2}- \left(x – y\right)^{2}$$

# Simplify the following expression by factorizing and cancelling

# $$\frac{9x^{2}-1}{6x^{2}+7x -3}$$

# $$ 4. Suppose that x-\frac{1}{x} and y- \frac{1}{y} and x\ne y $$

#  What is the value of $xy?$

**Section B: Indices and Surds**

# Indices:

# Which of these values is the odd one out?:

# 26

# 43

# 85/3

# 163/2

# 326/5

# Solve the following equations

# $$ a) 2^{x+1}=4^{x}$$

# $$ b) 3^{2x-1}=27$$

# $$ c) 5^{x^{2}+16}=625^{2x}$$

# $$ d) 4^{2x}=\frac{1}{8}$$

# 10x is a googol. What is the value of x and what is a googolplex?

# $2^{x}+ 3^{y}=209$. Find the values of the natural numbers x and y

# $x^{3}+ y^{3}+ z^{3}=853$. Find the values of the natural numbers x, y and z.

# Surds

# The result of each of the following takes the form 2x. Find the value of x

# √2 × 8

# √2 + √2 + √2 + √2

# (√8 × 4)3

# Expand and simplify the following

# (2 + √3)²

# (√2 + √3)²

# (√7 - √3)²

# (√2 + √8)²

# (√2 + √6)²

#  \*

**Section C: Sequences and Series**

# Complete the next two terms in these mathematical sequences

# 2 5 8 11 14

# 2 4 8 16 32

# 1 4 9 16 25

# 1 3 6 10 15

# 1 2 6 24 120

# 1 1 2 3 5 8

# 4 5 7 10 14

# 2 3 3.5 3.75

# 1 8 27 64 125

# 1 4 27 256 3125

# Complete the next two terms in these sequences. You might need some general knowledge to work some of them out

# 2 3 5 7 11 13

# 31 28 31 30 31

# 60 90 108 120

# 3 1 4 1 5

# 1:25 2:50 4:15 5:40 7:05

# 1 2 5 10 20

# 0001 0010 0011 0100 0101

# 0 0 1 2 67 62

# Series:

# A series is the sum of the terms of a sequence. The Greek letter ∑ (sigma) can be used to represent a series. The mathematical notation for the sum of the first n terms of a sequence is given as

# $$\sum\_{r=1}^{n}a\_{r}$$

# Where $a\_{r}$ is the rth term of the sequence?

# *Example 1*

# $$Calculate \sum\_{r=1}^{5}1, \sum\_{r=1}^{5}r, \sum\_{r=1}^{5}(2r+3)$$

# $$\sum\_{r=1}^{5}1=1+1+1+1+1=5 (Summing the first five terms) $$

# $$\sum\_{r=1}^{5}r=1+2+3+4+5=15 (Summing the first five terms) $$

# $$\sum\_{r=1}^{5}(2r+3)=5+7+9+11+13=45 (Summing the first five terms) $$

# There are some standard formulae that can be used to work out these sums

# $$The sum of n constant values: \sum\_{r=1}^{n}c=cn$$

# $$The sum of the first n natural numbers: \sum\_{r=1}^{n}r= \frac{n(n+1)}{2} $$

# $$\sum\_{r=1}^{n}\left(ar+b\right)=a\sum\_{r=1}^{n}r+\sum\_{r=1}^{n}b =\frac{an(n+1)}{2}+bn $$

# *Example 2*

# $$Calculate \sum\_{r=1}^{5}1, \sum\_{r=1}^{5}r, \sum\_{r=1}^{5}(2r+3)$$

# $$\sum\_{r=1}^{n}1=n \sum\_{r=1}^{5}1=5 $$

# $$\sum\_{r=1}^{n}r= \frac{n(n+1)}{2} \sum\_{r=1}^{5}r= \frac{5(5+1)}{2}=15 $$

# $$\sum\_{r=1}^{5}(2r+3)=2 ×15+3 ×5=45 $$

*Example 3*

# $$Find an expression in terms of n for \sum\_{r=1}^{n}(r+2\}$$

# $$\sum\_{r=1}^{n}(r+2)= \frac{n(n+1)}{2}+2n= \frac{n^{2}+n+4n}{2}= \frac{n(n+5)}{2} $$

# Find the values of the following

# $$a) \sum\_{r=1}^{8}2$$

# $$b) \sum\_{r=1}^{10}r$$

# $$ c) \sum\_{r=1}^{8}3r$$

# $$d) \sum\_{r=1}^{12}(2r-5)$$

# Find an expression in terms of n for the following

# $$a) \sum\_{r=1}^{n}3$$

# $$b) \sum\_{r=1}^{n}3r$$

# $$c) \sum\_{r=1}^{n}(2r-1)$$

# $$5. \sum\_{r=1}^{n}\left(ar+b\right)= n\left(2n+5\right) Find a and b$$

**Section D: Miscellaneous**

# Put the results of the following questions in order from smallest to largest

# The number of cm in a foot

# The number of grams in an ounce

# The mean of the prime numbers between 20 and 40

# The median of the first 10 square numbers

# The mode of the number of days in a month

# The circumference of a circle with radius 5 units

# The area of a rhombus with base 8 units and height 3.8 units

# The perimeter of a rhombus with base 8 units and height 3.8 units

# The surface area of a cube with side length 2.3 units

# The volume of a cube with side length 3.1 units

1. Multiply 142857 by 2, 3, 4, 5, 6 and 7. What do you notice?
2. Image you had two buckets, one of which holds 5 litres of water and one 3 litres.

You can fill either buckets up from the nearby river and empty it into a pond or fill the bucket up from the pond and throw the water back into the river. How could you end up with exactly 1 litre of water in the pond?

Empty 2 times 5 litres of water into the pool and remove 3 times 3 litres

2 × 5 – 3 × 3 = 1

or empty 2 times 3 litres into the pool and remove 1 times 5 litres.

 2 × 3 – 1 × 5 = 1

Repeat the question for the following scenarios

1. 5 and 7 litre buckets
2. 11 and 17 litre buckets
3. 12 and 15 litre buckets

**Section E: Probability**

# Two unbiased dice are rolled, one after the other.

# What is the probability that the value on the first die is greater than 4?

# What is the probability that value on the second die is odd?

# What is the probability that the values on both dice are the same?

# What is the probability that the two values are different?

# What is the probability that both values are even?

# What is the probability that the value on the second die is greater than that on the first?

# What is the probability that the sum of the two values equals 2?

# What is the probability that the sum of the two values equals 7?

# A bag contains red, yellow, green and purple marbles. When a marble is drawn from the bag it is not replaced. At the beginning of each question there are 3 red, 3 yellow, 3 green and 3 purple marbles in the bag.

# If 1 marble is drawn from the bag, what is the probability that it is red?

# If 2 marbles are drawn from the bag, what is the probability that they are the same colour?

# How many marbles should be drawn from the bag to ensure two marbles of the same colour are drawn?

# How many marbles should be drawn from the bag to ensure at least one marble of each colour is drawn?

# 3 marbles are drawn from the bag. What is the probability that none of them are purple?

# Two girls and three boys sit down of a row of 5 chairs. If the order they sit in is completely random

# What is the probability that no two boys are sitting together?

# What is the probability that the two girls are sitting next to each other?

# What is the probability that two boys are sitting together but not all three?

**Section F: Pythagoras’ Theorem**

# Find the length of the shortest side of a right-angled triangle if the lengths of the other two sides are 12 and 13.

# Find the hypotenuse (in surd form) of a right-angled triangle if the lengths of the other two sides are (2 + 2√3) and (4 - √3)

# A cuboid has sides of length 3cm, 5cm and 7cm. Calculate the length of the diagonal between the furthest two vertices.

**Pythagorean Triple**

# (a, b, c) is a Pythagorean triple if a, b and c are integers and a² + b² = c². (3, 4, 5) and (5, 12, 13) are two Pythagorean triplets that you may already know. There is actually an infinite number of Pythagorean triples.

# Consider the square of any odd number: for example 7² = 49

# Any odd number can be expressed as the sum of two consecutive integers: 49 = 24 + 25

# Using the result from question 4a: $\left(x+y\right)\left(x-y\right)=x^{2}-y²$

# 49 = (25 + 24)(25 – 24) = 25² - 24²

# So 7² + 24² = 25² which means that (7, 24, 25) is a Pythagorean triple.

# Using this method, find three more Pythagorean triples using the squares

# 9

# 11

# 13

#

**Section G: Prime Numbers**

The prime numbers are the natural numbers greater than 1 that have just two factors, 1 and itself.

**Prime Sieve**

The prime numbers up to a value n can be found using the following process,

Step 1: Create a list of all the numbers from 2 to n.

Step 2: Remove all the multiples of 2 that are greater than 2.

Step 3: Assign p to the next value in the list that has not been removed.

Step 4: Remove all the multiples of p that are greater than p.

Step 5: Repeat Step 3 and Step 4 until p is greater than √n

Any value that has not been removed is prime. This is known as a prime sieve.

1. In the grid below
	1. Remove all of the multiples of 2 but not 2 itself
	2. Then remove all of the multiples of 3 but not 3 itself
	3. Then remove all of the multiples of 5 but not 5 itself
	4. Then remove all of the multiples of 7 but not 7 itself
	5. Write down the remaining values.

.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** |
| **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** |
| **51** | **52** | **53** | **54** | **55** | **56** | **57** | **58** | **59** | **60** |
| **61** | **62** | **63** | **64** | **65** | **66** | **67** | **68** | **69** | **70** |
| **71** | **72** | **73** | **74** | **75** | **76** | **77** | **78** | **79** | **80** |
| **81** | **82** | **83** | **84** | **85** | **86** | **86** | **88** | **89** | **90** |
| **91** | **92** | **93** | **94** | **95** | **96** | **97** | **98** | **99** | **100** |

* 1. Are these all prime?
	2. Why is it not necessary to go beyond the value of √n?

**Goldbach Conjecture**

Every even number can be expressed as the sum of two even numbers. This appears to be true but has not been proved for every case.

For example 56 = 19 + 37 which are both primes.

1. Find pairs of primes that sum up to the following even numbers
2. 100
3. 102
4. 104
5. 106
6. 108
7. 110
8. Every natural number can be expressed as a product of prime factors. E.g. 24 = 2 × 2 × 2 × 3. Express the following values as a product of prime factors
9. 231
10. 364
11. 899
12. 1083
13. 1702
14. 142857
15. What is the smallest prime number that can be expressed as the sum of two prime numbers and also as the sum of three prime numbers?

**Mersenne Primes**

A Mersenne prime is a prime number of the form 2p – 1

For example

3 is a Mersenne prime as 22 – 1 = 3 and 3 is prime

7 is a Mersenne prime as 23 – 1 = 7 and 7 is prime

1. Find the next three Mersenne primes

**Section H: Squares**

1. How many ways can the number 89 be expressed as the sum of 3 non-zero square numbers?

# Simplify these expressions by expanding the brackets

#  (x + 3)²

#  (a + b + c)²

#  (x + y)² - (x – y)²

# Sums of Odd Numbers

# Add together the first 5 odd numbers

# Add together the first 6 odd numbers

# Add together the first 7 odd numbers

# What do you notice?

# Multiply together 4 consecutive numbers and add 1

# What do you notice? (Try several sets of numbers if this is not immediately obvious)

# Can you prove this is always that case when the first number is positive?

# Find the six possible digits that a square number can end in.

# Complete the numerical crosswords given that all of the answers are squares, no answer starts with a 0 and all of the answers in a single crossword are different.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Example

|  |  |  |
| --- | --- | --- |
| 1 | 4 | 4 |
| 9 |  | 9 |
| 6 | 4 |  |

 | a)

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

 | b)

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  | 8 |  |

 |  c)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  | 9 |  |
|  |  |  |  |

 |

 **Series**

# In Section D, you were given two standard formulae for summing n constant values and the first n natural numbers.

# $$\sum\_{r=1}^{n}1=n, \sum\_{r=1}^{n}r= \frac{n(n+1)}{2}$$

# Here is an additional formula for summing the first n square numbers

# $$ \sum\_{r=1}^{n}r²= \frac{n(2n+1)(n+1)}{6}$$

**Questions**

1. What is the sum of the first six square numbers?
2. Use the formula to calculate the result
3. Add up the values to check that the formula has given you the correct value
4. Find the values of the following

# $$a) \sum\_{r=1}^{4}r²$$

# $$b) \sum\_{r=1}^{5}4r²$$

# $$ c) \sum\_{r=1}^{6}(r^{2}-2r)$$

# Find an expression in terms of n for

# $$a) \sum\_{r=1}^{n}2r²$$

# $$b) \sum\_{r=1}^{n}(r^{2}-1)$$

# $$c) \sum\_{r=1}^{n}(r^{2}+r)$$

1. Substituting in n-1 for n gives the sum of the first n-1 square numbers

# $$ \sum\_{r=1}^{n-1}r²= \frac{(n-1)(2n-1)n}{6}$$

Simplify the following expression and explain your result

$$\sum\_{r=1}^{n}r^{2}- \sum\_{r=1}^{n-1}r^{2} $$

# Section I: Complex Numbers

# The square root of -1

# You may have been told that it is not possible to find the root of a negative number. If you try to find the value of $\sqrt{-3}$ using a calculator, you will usually be presented with a “Math Error”.

# However, it is possible to solve this problem by defining i to be equal to $\sqrt{-1}$ . This number is known as an imaginary number.

# Using i, an expression can be written for the square root of any negative number.

# *Example1*: Find an expression for the square root of -4 and that of -7 in terms of i

# $\sqrt{-4}$ = $\sqrt{4}$ × $\sqrt{-1}$ = 2$i$

#  $\sqrt{-7}$ = $\sqrt{7}$ × $\sqrt{-1}$ = $\sqrt{7}i$

# A number can be made up of a real and an imaginary part, e.g. 5 + 4i. 5 is the real part of the number and 4i is the imaginary part. These numbers are known as complex numbers.

# Complex numbers result from the solution of a quadratic equation where the discriminant, (b² - 4ac), is negative.

#  Complex roots of quadratic equations

# *Example 2*: Solve x² + 2x + 5 = 0.

# Using the quadratic formula with the values: a=1, b=2, c=5

# $$x=\frac{-2 \pm \sqrt{2²-4×1 ×5}}{2 ×1}$$

# $$x=\frac{-2 \pm \sqrt{-16}}{2 }$$

# $$x=\frac{-2 \pm 4i}{2 }$$

# $$x= -1 \pm 2i$$

# Operations on Complex Numbers

# Complex numbers can be added together by adding their real and complex parts individually.

# *Example 3*: Add the complex numbers (4 + 5i) and (2 - i)

#  (4 + 5i) + (2 - i) = 6 + 4i

Complex numbers can be multiplied together. Note that since i = $\sqrt{-1}$ then i × i = -1.

# *Example 4*: Find 5i × 6i

#  5i × 6i = 30 × (-1)

# 5i × 6i = -30

# *Example 5*: Find (3 + 2i) × (4 + 5i)

# 3 × (4 + 5i) = 12 + 15i

# 2i × (4 + 5i) = 8i - 10

# (3 + 2i) × (4 + 5i) = (12 + 15i) + (8i - 10)

# (3 + 2i) × (4 + 5i) = 2 + 23i

# Questions:

# Write the following negative roots as an expression in terms of i

|  |
| --- |
| $\sqrt{-5}$$\sqrt{-9}$$\sqrt{-121}$$\sqrt{-8}$  |

# Solve the following equations

# $$ a) x² + 2x + 10 = 0$$

# $$ b) x^{2}- 3x + 4 = 0 $$

#  $c) 2x² - x + 3 = 0 $

# Find the values of the following

# (2 + i) + (4 + 5i)

# (3 + 2i) + (5 - 10i)

# 4 × 3i

# 6i × -2

# 3i × 5i

# 3 × (4 + i)

# 3i × (4 + i)

# (2 + i) × (4 + 3i)

# (2 - i)( (4 + 3i)

# (5 - 2i) × (2 - 3i)

**Section J: Roots of Polynomials**

# Roots of a quadratic equation

# A quadratic equation always has two roots, both of which may be equal. The roots can be real of complex.

# Let the roots be $α$ (alpha) and $β$ (beta)

# $$\left(x- α\right)\left(x-β\right)=x^{2}-\left(α+ β\right)x+ αβ$$

# So for the quadratic equation $ax^{2}+bx+c=0$,

# $$\left(α+ β\right)= -\frac{b}{a} and αβ= \frac{c}{a}$$

# *Example 1:*

# Find the sum and the product of the roots of the quadratic equation $x^{2}+7x+11=0$

# The sum of the roots $(α+ β)$ = -7

# The product of the roots $αβ$ = 11

# *Example 2:*

# Find the sum and the product of the roots of the quadratic equation $2x^{2}+7x+11=0$

# The sum of the roots $(α+ β)$ = -3.5

# The product of the roots $αβ$ = 5.5

# *Example 3:*

# $α$ and $β$ are the two roots of the quadratic equation $x^{2}-5x+7=0$.

# Find the value of $α^{2}+ β²$.

# $α+ β$ = 5 $αβ$ = 7

# $α^{2}+ β²$ = $\left(α+ β\right)^{2}-2αβ$ = 25 – 14 = 11

# *Example 4:*

# One root of the quadratic equation $x^{2}-11x+k=0$ is 5. Find the other root and the value of k.

# Let the other root be $β$

#  5 + $β$ = 11 $β$ = 6

#  k = 5 × 6 = 30 $x^{2}-11x+30=0$

Questions

# Find the sum and the product of the roots of the following equations without solving them first

# $x^{2}-5x+14=0$

# $2x^{2}+3x+8=0$

# $5x^{2}+x-16=0$

# For each of the above, find the values of $3αβ$, $\left(α+ β\right)^{3}$ and $α^{2}+ β²$

# One root of the quadratic equation $x^{2}+px+8=0$ is 4. Find the other root and the value of p

# One root of the quadratic equation $x^{2}-8x+q=0$ is 3 times the value of the other. Find the roots and the value of q

# The quadratic equation $x^{2}+px+q=0$ has equal roots. Find an expression for q in terms of p.

# Roots of a cubic equation

# A cubic equation always has three roots, two or all three of which may be equal. The roots can be real or complex.

# Let the roots be $α$ (alpha), $β$ (beta) and $γ$ (gamma)

# $$\left(x- α\right)\left(x-β\right)\left(x- γ\right)=x^{3}-\left(α+ β+γ \right)x²+\left( αβ+ αγ+ βγ\right)x+αβγ $$

# So for the quadratic equation $ax^{3}+bx^{2}+cx+d=0$

# $$\left(α+ β+ γ\right)= -\frac{b}{a}$$

# $$αβ+αγ+βγ= \frac{c}{a}$$

# $$αβγ= -\frac{d}{a}$$

# *Example 1:*

# Find the sum and the product of the roots of the cubic equation $x^{3}+2x^{2}+3x+4=0$

# The sum of the roots $\left(α+ β+ γ\right)$ = -2

# The product of the roots $αβγ$ = 4

# *Example 2:*

# $α$, $β$ and $γ$ are the three roots of the quadratic equation $x^{3}+2x^{2}+3x+4=0$

# Find the value of $α^{2}+ β²+ γ²$.

# $\left(α+ β+ γ\right)= -2$

# $αβ+αγ+βγ=3$

# $α^{2}+ β²+ γ²$.= $\left(α+ β+ γ\right)^{2}-2(αβ+ αγ+βγ)$ = 4 – 6 = -2

# Questions

# Find the sum and the product of the roots for the following equations without solving them first

# $x^{3}+3x^{2}-7x+22=0$

# $5x^{3}-2x^{2}+6x+11=0$

# $4x^{3}+5x^{2}-x-5=0$

# Find the values of p, q and r for a cubic equation $x^{3}+px^{2}+qx+r=0$ with the following roots.

# 1, 2 and 3

# 2, -7 and 11.

# 2.5, 4 and -3.5

# For each of the above, find the value of ($α^{2}+ β²+ γ²$)

# Two of the roots of the cubic equation $x^{3}-2x^{2}+px+q=0$ are 3 and -4. Find the other root and the values of p and q

# One root of the cubic equation $x^{3}-10x^{2}+px-30=0$ is 2. Find the other two roots and the value of p.